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Imen Nasr, Leila Najjar Atallah, Sofiane Cherif, Benoit Geller, Jianxiao Yang

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Imen Nasr [1,2], Leïla Najjar Atallah [1], Sofiane Cherif [1], Benoît Geller [2] and Jianxiao Yang [2]

[1] *Ecole Supérieure des Communications de Tunis Sup'Com, Tunisia*

[2]*Ecole Nationale Supérieure de Techniques Avancées ENSTA ParisTech, France*

[1]{nasr.imen,leila.najjar,sofiane.cherif}@supcom.rnu.tn

[2]{imen.nasr,benoit.geller,jianxiao.yang}@ensta-paristech.fr

***Abstract*—**[1] **Time delay synchronization is crucial for the reception quality in digital transmission systems. In this contribution, we consider a maximum likelihood approach and incorporate a soft-demapper to improve the synchronization performance. In particular, the proposed scheme allows to update the time delay at each symbol with an adaptive loop using the Log-Likelihood Ratio (LLR) of each bit provided by the demapper. Simulation results show that the proposed approach provides improvements compared to non data aided approach while avoiding data aided approach overhead.**

I. Introduction

Time and carrier phase synchronization is one of the most important features in a communication system receiver to be efficiently performed prior to detection and decoding in order to correcly recover the transmitted data. The main issue in a synchronization system is to work properly even at very low SNR and this is a hard task as it is processed at the front end. In this paper, we focus on time delay synchronization. Several Data Aided (DA) and Non Data Aided (NDA) time delay estimation techniques such as [1] and [2] have been proposed in the literature. However, in a DA mode, pilot signals are needed which leads to the increase of the overhead in the communication network and reduces the spectrum efficiency. In the NDA mode, some statistical information about the transmitted signal are lost and this deteriorates the system performance especially in bad channel conditions. Indeed, deriving good reference signals may improve the performance while preserving a high spectral efficiency.

Iterative receivers are able to compute soft information about the received symbols which helps the synchronizer to better estimate the time delay. This technique has been already used for phase estimation in turbo decoding receivers [3]. It was also exploited in [4] with an expectation maximization algorithm in the Maximum Likelihood (ML) synchronization framework. In [5]–[7], the authors derive the Bayesian and hybrid Cramer-Rao bounds (BCRB and HCRB) for the code aided (CA), the DA, and the NDA dynamical phase estimation of QAM modulated signals and theoretically show the possible improvement brought by a soft CA technique. In the reference [8], a forward backward algorithm was proposed to cope with the performance deterioration in the NDA mode. Although this technique has shown better performance than the CA approach, it can only be implemented in an off-line transmission system.

In this paper, we are interested in deriving a time delay estimation algorithm which is based on the maximization of the likelihood function with the use of soft demapping information.

This paper is organized as follows. In section II, the system model and the time delay estimation based on the ML approach are presented. In section III, the derivation of the DA and the NDA algorithms are first given and then, the new soft algorithm for the time delay estimation is derived. Simulation results are provided in section IV and confirm our analysis. The last section concludes our work.

II. System Model

Let us consider the transmitted signal $s(t)$ written as:

$$s(t) = \sum_{i=-\infty}^{+\infty} a_i h(t - iT), \qquad (1)$$

where a_i denotes the BPSK transmitted symbols, $h(t)$ is the impulse response of the transmission filter and T is the symbol period.

The received signal is

$$r(t) = s(t - \tau) + n(t), \qquad (2)$$

where the channel introduces a random delay τ to the transmitted signal $s(t)$. In (2) the received signal is disturbed by an additive white Gaussian noise (AWGN) $n(t)$ of zero mean and of variance σ^2.

It is worth noting that without loss of generality, the considered model can also be applied to multipath and fading channels, in which case $h(t)$ is the convolution of the transmission filter and of the channel impulse response. The time delay is estimated in the maximum likelihood sense by maximizing the likelihood function according to the following equation [9]

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$$\hat{\tau} = \arg\max_{u} \{\Lambda(u,a)\}, \tag{3}$$

where a is the vector of the transmitted symbols and

$$\Lambda(u,a) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2\sigma^2}\int_{T_0} |r(t) - s(t-u)|^2 \, dt\right), \tag{4}$$

is the likelihood function and T_0 is the observation interval. Equivalently, the log-likelihood function $\Lambda_L(u,a)$ can be used instead of $\Lambda(u,a)$, where

$$\Lambda_L(u,a) = -\frac{1}{2\sigma^2}\int_{T_0} |r(t) - s(t-u)|^2 \, dt + \ln\left(\frac{1}{2\pi\sigma^2}\right). \tag{5}$$

Given that $\int_{T_0} |r(t)|^2 dt$ is independent of u and $\int_{T_0} |s(t-u)|^2 \, dt$ represents the transmitted signal energy which is independent of u (when T_0 is large enough), one has

$$\Lambda_L(u,a) = \frac{1}{\sigma^2}\mathrm{Re}\left(\int_{T_0} r(t)s^*(t-u)dt\right) + C_1, \tag{6}$$

where $(.)^*$ is the complex conjugate and C_1 is a constant term independent of u.

A further development of the log-likelihood function is given by the following equations

$$\begin{aligned}
\Lambda_L(u,a) &= \mathrm{Re}\left\{\frac{1}{\sigma^2}\int_{T_0} r(t)s^*(t-u)dt\right\} \\
&= \mathrm{Re}\Bigg\{\frac{1}{\sigma^2}\int_{T_0}\left(\sum_i a_i h(t-iT-\tau) + n(t)\right) \\
&\qquad \left(\sum_j a_j h(t-jT-u)\right)^* dt\Bigg\} \\
&= \mathrm{Re}\Bigg\{\sum_{i,j}\frac{a_j^* a_i}{\sigma^2}\int_{T_0} h(t-iT-\tau)h^*(t-jT-u)dt \\
&\qquad + \sum_j \frac{a_j^*}{\sigma^2}\int_{T_0} h^*(t-jT-u)n(t)dt\Bigg\},
\end{aligned} \tag{7}$$

where we dropped the term independent of u denoted by C_1 in (6).

Let us consider

$$\begin{aligned}
g(t) &= h(t) \otimes h^*(-t), && (8)\\
y_j(u) &= \sum_i a_i g\left((j-i)T - (\tau - u)\right), && (9)\\
v_j(u) &= \int_{T_0} h^*(t-jT-u)n(t)dt, && (10)\\
x_j(u) &= y_j(u) + v_j(u), && (11)
\end{aligned}$$

where $\otimes$ denotes the convolution operation.

$x_j(u)$ is seen as the matched filter output of the received signal, $y_j(u)$ is the useful part and $v_j(u)$ is a colored gaussian noise. Then, the log-likelihood function becomes

$$\Lambda_L(u,a) = \frac{1}{\sigma^2}\mathrm{Re}\left(\sum_j a_j^* x_j(u)\right). \tag{12}$$

This is why, the estimate in the maximum likelihood sense is obtained through the following equation

$$\hat{\tau} = \arg\max_{u} \mathrm{Re}\left(\sum_j a_j^* x_j(u)\right). \tag{13}$$

Since it is difficult in practice to maximize the previous equation with respect to u analytically, adaptive algorithms such that proposed by Mueller & Muller (M&M) [1] are implemented whose objective is to converge iteratively to the desired value of the parameter. From the classical ML approach and the M&M algorithm, we hereafter derive a new time delay estimation technique which uses the soft demapping information from the decoder block. The new algorithm derivation is going to be presented in the next paragraph.

III. Feedback symbol timing detector

In this part, we are interested in deriving the Time Error Detector (TED) equation that iteratively maximizes the log-likelihood function (13). The traditional block diagram is given in Fig. 1.

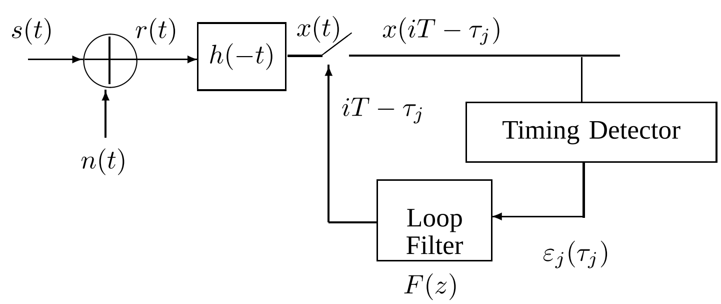

Fig. 1. Tracking loop description

The tracking loop is given by the following equation

$$\tau_j = \tau_{j-1} + \varepsilon_j(\tau_{j-1}) \otimes l_j^f, \tag{14}$$

where $\varepsilon_j(\tau_{j-1})$ is the updating error given by the timing detector and l_j^f is the loop filter corresponding impulse response coefficients.

By considering $F(z)$ the transfer function of the loop filter (the Z-transform of l_j^f), the transfer function from τ_{j-1} to τ_j is

$$G(z) = \frac{F(z)}{z - 1 + F(z)}. \tag{15}$$

A loop filter for which the steady state error vanishes can be chosen as [10]

$$F(z) = \beta, \tag{16}$$

where $0 < \beta < 1$. This leads to:

$$\tau_j = \tau_{j-1} + \beta\varepsilon_j(\tau_{j-1}). \tag{17}$$

A. Data Aided M&M Algorithm

According to equation (13), $x_j(\hat{\tau})$ should tend towards a_j in absence of noise and synchronization errors. Thus, by considering the symbol error $e_j = x_j(u) - a_j$ and

$$\begin{aligned} J(u) &= \sum_j |e_j|^2 && (18) \\ &= \sum_j |a_j|^2 + \sum_j |x_j(u)|^2 && (19) \\ &\quad -2\sum_j \text{Re}(a_j^* x_j(u)), \end{aligned}$$

where the summation is carried along the observation interval, the maximization of (13) is equivalent to the minimization of $J(u)$ with respect to u.
We can restrict our study to the minimization of the instantaneous error $|e_j|^2$ at any time index j. Differentiating $|e_j|^2$ with respect to u gives:

$$\frac{\partial |e_j|^2}{\partial u} = (x_j(u) - a_j)\frac{\partial x_j^*(u)}{\partial u} + \left(x_j^*(u) - a_j^*\right)\frac{\partial x_j(u)}{\partial u}, \tag{20}$$

where

$$\frac{\partial x_j(u)}{\partial u} = \sum_i a_i \frac{\partial g\left((j-i)T - (\tau - u)\right)}{\partial u} + \frac{\partial v_j(u)}{\partial u}. \tag{21}$$

According to [11], considering that $\frac{\partial g(t)}{\partial t} = 0$ around 0 and that it is equal to C and $-C$ respectively in the vicinity of $-T$ and $+T$, where C is a constant, in addition to neglecting the contributions of $\frac{\partial g(t)}{\partial t}$ beyond $\pm T$ and assuming that the error $u - \tau$ is very small, the tracking loop equation is given by:

$$\tau_j = \tau_{j-1} + \mu \text{Re}\left(a_j^* x_{j-1}(\tau_{j-1}) - a_{j-1}^* x_j(\tau_{j-1})\right), \tag{22}$$

where μ is the step size. For better robustness in a non-stationary context, this step size can be made adaptive as in [12] and [13], however for simplicity reasons we here choose a small constant value for μ.
This timing error detector was also derived by Mueller & Muller [1].

B. Non Data Aided M&M Algorithm

In order to avoid the transmission of long training sequences causing a spectral efficiency reduction, a NDA delay tracking algorithm is obtained by replacing the true transmitted data a_j in the previous developments with its estimated value $\hat{a}_j$; $\hat{a}_j$ is obtained by making a hard decision on the received symbols. The updating equation then becomes:

$$\tau_j = \tau_{j-1} + \mu \text{Re}\left(\hat{a}_j^* x_{j-1}(\tau_{j-1}) - \hat{a}_{j-1}^* x_j(\tau_{j-1}))\right), \tag{23}$$

where

$$\hat{a}_j = \text{sign}\left(x_j(\tau)\right). \tag{24}$$

This timing detector is used with a BPSK or QPSK modulation.

C. Soft M&M Algorithm

Instead of making unreliable hard decisions on the received data (NDA approach) to still optimize the spectral efficiency, we avoid sending preamble sequences (DA approach) and propose to take advantage of the output of the system decoder to enhance the time recovery performance. Let us consider, λ_j the output of the soft demodulator at any time index j.
For a BPSK modulated signal, we know that:

$$p(a_j = \pm A) = \frac{\exp\left(\pm\frac{\lambda_j}{2}\right)}{2\cosh\left(\frac{\lambda_j}{2}\right)}, \tag{25}$$

and

$$\Lambda_L(u,a) = \exp\left(\frac{1}{\sigma^2}\sum_{j=1}^{N} a_j \tilde{x}_j(u)\right), \tag{26}$$

where

$$\tilde{x}_j(u) = \text{Re}\left(x_j(u)\right) \tag{27}$$

By averaging the likelihood function over the data a_j

$$\begin{aligned} \Lambda(u) &= \prod_j \frac{1}{2\sigma^2}\Bigg[\frac{\exp(\frac{\lambda_j}{2})}{\cosh(\frac{\lambda_j}{2})}\exp\left(\frac{1}{\sigma^2}A\tilde{x}_j(u)\right) + \\ &\quad \frac{\exp(\frac{-\lambda_j}{2})}{\cosh(\frac{\lambda_j}{2})}\exp\left(-\frac{1}{\sigma^2}A\tilde{x}_j(u)\right)\Bigg] && (28) \end{aligned}$$

$$\Lambda(u) = \prod_j \frac{\cosh\left(\frac{\lambda_j}{2} + \frac{1}{\sigma^2}A\tilde{x}_j(u)\right)}{\sigma^2\cosh(\frac{\lambda_j}{2})} \tag{29}$$

The log-likelihood function becomes

$$\Lambda_L(u) = \sum_j \ln\left(\frac{\cosh\left(\frac{\lambda_j}{2} + \frac{1}{\sigma^2}A\tilde{x}_j(u)\right)}{\cosh(\frac{\lambda_j}{2})}\right) + C_2, \tag{30}$$

where C_2 is a constant term. By differentiating $\Lambda_L(u)$ with respect to u we obtain:

$$\frac{\partial \Lambda_L(u)}{\partial u} \propto \sum_j \frac{A}{\sigma^2}\frac{\partial \tilde{x}_j(u)}{\partial u}\tanh\left(\frac{\lambda_j}{2} + \frac{1}{\sigma^2}A\tilde{x}_j(u)\right). \tag{31}$$

Unlike the NDA approach which replaces a_j in (22) by the hard decision, we propose to replace the data a_j by the soft symbol

$$\tilde{a}_j = \tanh\left(\frac{\lambda_j}{2} + \frac{1}{\sigma^2}A\tilde{x}_j(\tau_{j-1})\right). \tag{32}$$

In the next section simulation results of the proposed technique are presented.

IV. SIMULATION RESULTS

We simulate the case of a BPSK signal with an up-sampling factor equal to 8, passed through a raised cosine filter, with a roll-off factor α. Results are given for 100 symbol blocks averaged over 400 Monte Carlo iterations. The LLRs are calculated with a soft demapping of the received signal. τ is initialized to 0 and its estimated value is depicted at the end of the block when the steady state is achieved. We assume having a new symbol $x_j(\tau)$ at each iteration and the value of $x_j(\tau_{j-1})$ is obtained via a quadratic interpolation.

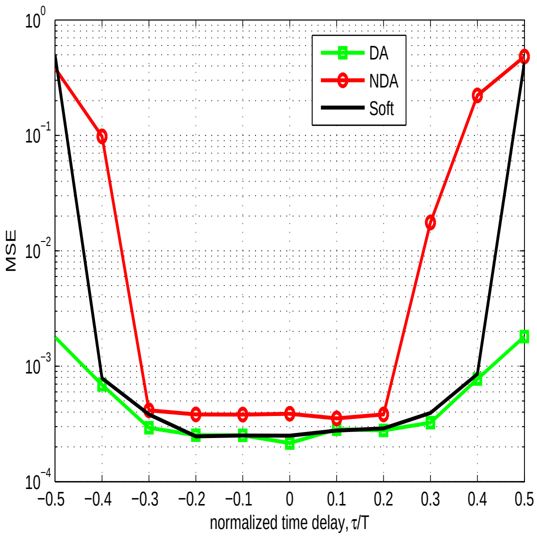

Fig. 2. MSE vs normalized time delay, BPSK signal, SNR=0 dB, $\alpha = 0.3$

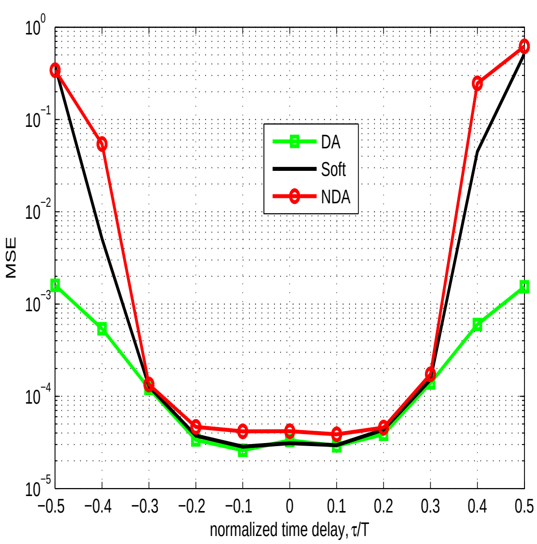

Fig. 3. MSE vs normalized time delay, BPSK signal, SNR=10 dB, $\alpha = 0.3$

Fig. 2 and 3 display the mean square error (MSE) in terms of the normalized delay τ/T for an SNR which is equal to 0dB

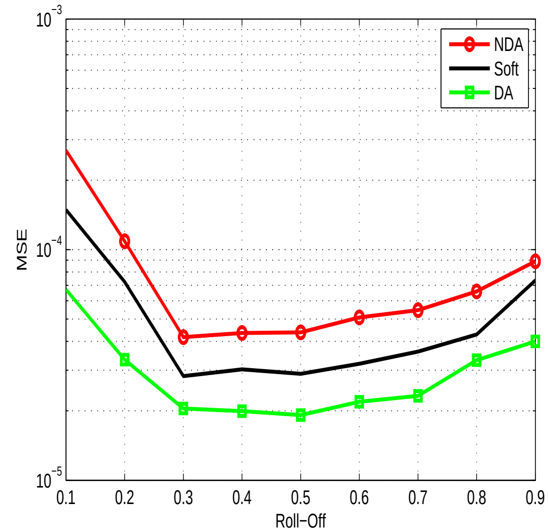

Fig. 4. MSE vs Roll-Off factor, BPSK signal, SNR=10 dB, normalized time delay=0.1

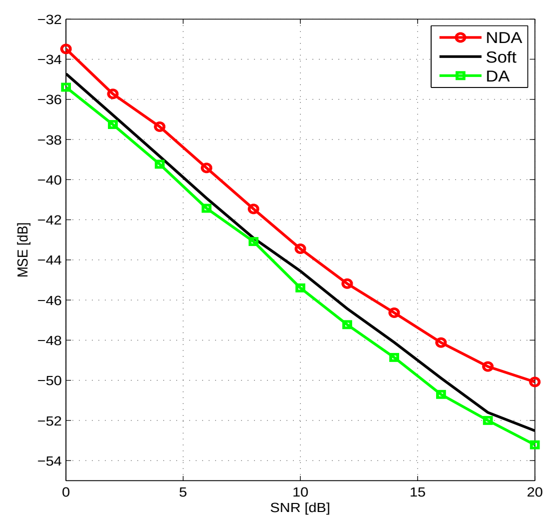

Fig. 5. MSE vs SNR, BPSK signal, normalized time delay=0.1, $\alpha = 0.5$

and 10dB respectively. A first ascertainment is that compared to the NDA mode, the MSE is decreased by exploiting the soft information related to the symbols. This is particularly true at low SNR for which modern systems are constrained to work. The DA mode still achieves the best performances however it leads to a higher loss of spectral efficiency due to the transmission of pilot symbols. For instance, for $\tau/T = 0.1$ and a SNR equal to 0dB the MSE is equal to 3×10^{-4} for the DA mode, 3.2×10^{-4} for the proposed technique and 5×10^{-4} for the NDA mode.

Fig. 4 shows the mean square error in terms of the roll-off factor α for a SNR$=10$dB. This figure confirms the previous results. Indeed, for any value of the roll-off factor, the soft algorithm still has better performance than the NDA mode. Similar and even better results could be obtained if the soft information is provided by a soft channel decoder instead of a demapper; however the results would be obtained at the cost of a larger complexity (decoder's architecture, number of iterations).

Fig. 5 presents the mean square error as a function of the SNR for τ/T equal to 0.1 and $\alpha = 0.5$. This figure shows a decrease in the estimation error thanks to the introduction of soft symbols compared to the NDA mode, and this is especially appealing for low SNR values. In fact, for a MSE of -40dB the proposed technique allows a gain of 2dB compared to the NDA approach whereas the gap is only of about 0.5dB with respect to the DA approach.

V. Conclusion

In this paper we presented a Maximum Likelihood time delay recovery algorithm which uses some soft information. The algorithm updates the time delay estimate at each received symbol and this seems to be a promising way to deal with low SNR. This technique has shown better synchronization performance in comparison to the non data aided mode with no need for pilot signals. In some future work, better results are expected by calculating the soft information when powerful channel codes are used [14]–[16].

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